MATH 113 - WORKSHEET 2
WEDNESDAY 7/3

(1) Let G be a group, and let S be a set of generators for G. Show that G is abelian if and only if $\forall a, b \in S$, $ab = ba$.

(2) There is a group D_8 of order 8 with generators x and y such that $x^4 = e$, $y^2 = e$, and $xy = yx^3$. This group is called the *dihedral group of order* 8.
   (a) Show that every element of D_8 can be written in the form $y^i x^j$, with $i \in \{0, 1\}$ and $j \in \{0, 1, 2, 3\}$.
   (b) Writing elements in this standard form, compute $(yx)(yx)$ and $(yx^2)(y)$.

(3) There is a group Q of order 8 with generators i, j, k, and m, such that $ij = k$, $jk = i$, $ki = j$, $i^2 = j^2 = k^2 = m$, and $m^2 = e$. This group is called the *quaternion group*.
   (a) Show that m commutes with each of the other generators.
   (b) Show that $ji = mk$, $kj = mi$, and $ik = mj$.
   (c) Show that every element of Q can be expressed as one of $e, i, j, k, m, mi, mj, mk$.

(4) Show that every group of order 4 is abelian.
   Hint: Let $a, b \in G$. What are the possible values of ab and ba?

(5) Define the function $\#d : \mathbb{Z}^+ \to \mathbb{Z}^+$, where $\#d(n) =$ the number of divisors of n. For example, $\#d(6) = 4$, since the divisors of 6 are 1, 2, 3, and 6. For another example, $\#d(p) = 2$ for any prime number p, since the divisors of p are 1 and p.
   Let G be a finite cyclic group of order n. Show that G has exactly $\#d(n)$ subgroups.

(6) Consider the interval $[0, 1) \subseteq \mathbb{R}$ with the binary operation $+_c : [0, 1) \times [0, 1) \to [0, 1)$ of "addition mod 1". For example, $.1 +_c .42 = .52$, $.75 +_c .3 = .05$, and $.19 +_c .81 = 0$. $C = ([0, 1), +_c)$ is a group, called the *circle group*.
   Find an element of order 2 in C. Now for each $n \in \mathbb{Z}^+$, find an element of order n in C. Finally, find an element of infinite order in C.